Tips for Establishing Trigonometric Identities

Getting Started:

1. **Start by determining which side of the identity is simpler and which side is more complicated.** Rewrite the identity with the simpler side on the left and the more complicated side on the right. We will establish a trigonometric identity by working the right–hand side until it looks like the left. (Occasionally we may have to work the left side a little so that we can make sure both sides match.)
2. **On *both* sides of the equation express $\tan q$, $\csc q$, $\sec q$, and $\cot q$ in terms of $\sin q$ and $\cos q$.**

Algebraic Steps to Consider:

1. **Look at the number of terms on both sides**, then determine if you need to combine fractions or break up fractions so that the number of terms will match.
   - Combine two or more fractions over a common denominator to produce a single fraction.
   - Split up a single fraction with two or more terms in the numerator by putting each term in the numerator over the common denominator.
2. **Multiply the numerator and denominator of a fraction by a form of the number "1".**
   - For complex fractions multiply both the numerator and denominator by the LCD to clear fractions.
   - To match on a particular denominator you may need to multiply both the numerator and denominator by the denominator you are trying to get.
   - To rationalize the denominator, multiply both the numerator and denominator by the conjugate of the denominator
   - Multiply by either the conjugate of the numerator or denominator to get a Difference of Squares, so that we can use a Pythagorean Identity.
3. **Expand expressions by multiplication, if possible, then combine like terms and simplify.**
   - Expand a Binomial Squared. $(a \pm b)^2 = a^2 \pm 2ab + b^2$
   - Expand the Product of a Sum and Difference of Two Terms. $(a+b)(a-b) = a^2 - b^2$
4. **Factor the numerator and denominator, if possible, to cancel common factors taking the fraction to lowest terms.**
   - Factor out the Greatest Common Factor.
   - Factor a Difference of Squares. $a^2 - b^2 = (a+b)(a-b)$
   - Factor a Sum/Difference of Cubes. $a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$
5. **Recognize patterns from Identities and Formulas.**
   - Reciprocal, Quotient, Pythagorean, and Even/Odd Identities.
   - Sum and Difference Formulas.
   - Double/Half Angle Formulas and Power Reducing Formulas.
   - Product–to–Sum and Sum–to–Product Formulas.

Knowing Which Formula to Use:

1. There are no formulas (except Pythagorean Identities) involving $\csc q$, $\sec q$, and $\cot q$. If you are given one of these three functions apply a Reciprocal Identity then decide what to do next.
2. If the angle is…

$-q$	use	Even–Odd Identities
$a \pm b$	use	Sum and Difference Formulas
$2q$	use	Double Angle Formulas
$\frac{q}{2}$	use	Half Angle Formulas

3. If you have …

$\sin^2 q$, $\cos^2 q$, $\tan^2 q$, $\csc^2 q$, $\sec^2 q$, or $\cot^2 q$	use	Pythagorean Identities or a Power Reducing Formula
$\sin a \cos b$, $\sin a \sin b$, $\cos a \sin b$, or $\cos a \cos b$	use	Product–to–Sum Formulas
$\sin a \pm \sin b$ or $\cos a \pm \cos b$	use	Sum–to–Product Formulas